Constructing Polytopes via a Vertex Oracle

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Joint work with I.Z. Emiris, C. Konaxis (now U. Crete) and
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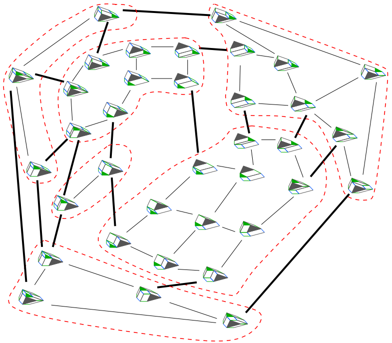
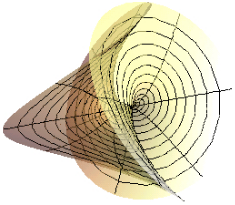
Main actor: resultant polytope

- Geometry: Minkowski summands of secondary polytopes, equival. classes of secondary vertices, generalize Birkhoff polytopes
- Motivation: useful to express the solvability of polynomial systems
- Applications: discriminant and resultant computation, implicitization of parametric hypersurfaces





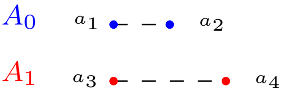
Enneper's Minimal Surface

Existing work

- Theory of resultants, secondary polytopes, Cayley trick [GKZ '94]
- TOPCOM [Rambau '02] computes all vertices of secondary polytope.
- [Michiels & Verschelde DCG'99] coarse equivalence classes of secondary polytope vertices.
- [Michiels & Cools DCG'00] decomposition of $\Sigma(\mathcal{A})$ in Minkoski summands, including $N(\mathcal{R})$.
- Tropical geometry [Sturmfels-Yu '08]: algorithms for resultant polytope (`GFan` library) [Jensen-Yu '11] and discriminant polytope (`TropLi` software) [Rincn '12].

What is a resultant polytope?

- Given $n+1$ point sets $A_0, A_1, \dots, A_n \subset \mathbb{Z}^n$



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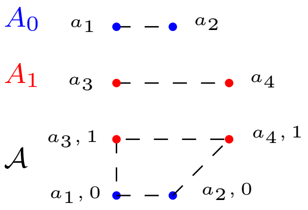
- Given $n+1$ point sets $A_0, A_1, \dots, A_n \subset \mathbb{Z}^n$
- $\mathcal{A} = \bigcup_{i=0}^{n} (A_i \times \{e_i\}) \subset \mathbb{Z}^{2n}$ where $e_i = (0, \dots, 1, \dots, 0) \subset \mathbb{Z}^n$



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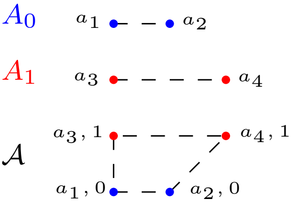
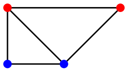
- Given T a triangulation of $conv(\mathcal{A})$, a cell is a-mixed if it contains 2 vertices from A_j, $j \neq i$, and one vertex $a \in A_i$.





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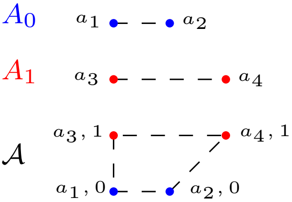
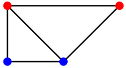
- $\rho_T(a) = \sum_{\substack{a-\text{mixed} \\ \sigma \in T : a \in \sigma}} \text{vol}(\sigma) \ \in \mathbb{N}, \quad a \in \mathcal{A}$





$$\rho_T = (0, 2, 1, 0)$$

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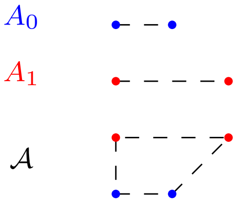
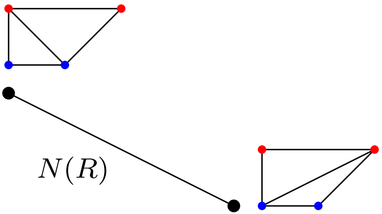
- Resultant polytope $N(R) = conv(\rho_T : T \text{ triang. of } conv(\mathcal{A}))$



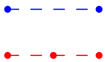


Connection with Algebra

- The support of a polynomial is the the set of exponents of its monomials with non-zero coefficient.
- The resultant R is the polynomial in the coefficients of a system of polynomials which is zero iff the system has a common solution.
- The resultant polytope $N(R)$, is the convex hull of the support of R.

A_0

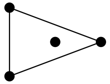
A_1



$$f_0(x) = ax^2 + b$$

$$f_1(x) = cx^2 + dx + e$$

$N(R)$



$$R(a,b,c,d,e) = ad^2b + c^2b^2 - 2caeb + a^2e^2$$

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A_0

$$f_0(x,y) = ax + by + c$$

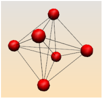
A_1

$$f_1(x,y) = dx + ey + f$$

A_2

$$f_2(x,y) = gx + hy + i$$

$N(R)$



4-dimensional Birkhoff polytope

$$R(a,b,c,d,e,f,g,h,i) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

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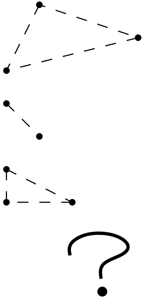


A_0 $\qquad$ $$f_0(x, y) = axy^2 + x^4y + c$$

A_1 $\qquad$ $$f_1(x, y) = dx + ey$$

A_2 $\qquad$ $$f_2(x, y) = gx^2 + hy + i$$

$N(R)$

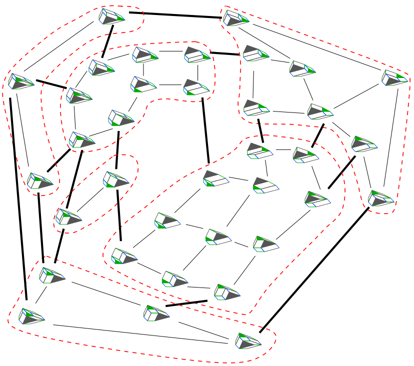
NP-hard to compute the resultant
in the general case

The idea of the algorithm

Input: $\mathcal{A} \in \mathbb{Z}^{2n}$ defined by $A_0, A_1, \ldots, A_n \subset \mathbb{Z}^n$
Simplistic method:

- compute the secondary polytope $\Sigma(\mathcal{A})$
- many-to-one relation between vertices of $\Sigma(\mathcal{A})$ and $N(R)$ vertices

Cannot enumerate 1 representative per class by walking on secondary edges

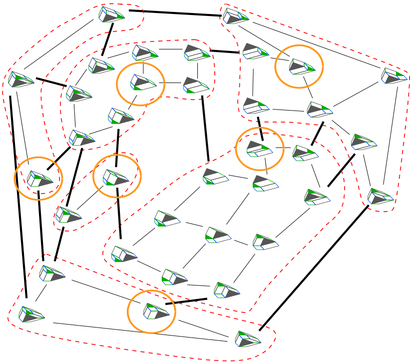


The idea of the algorithm

Input: $\mathcal{A} \in \mathbb{Z}^{2n}$ defined by $A_0, A_1, \ldots, A_n \subset \mathbb{Z}^n$

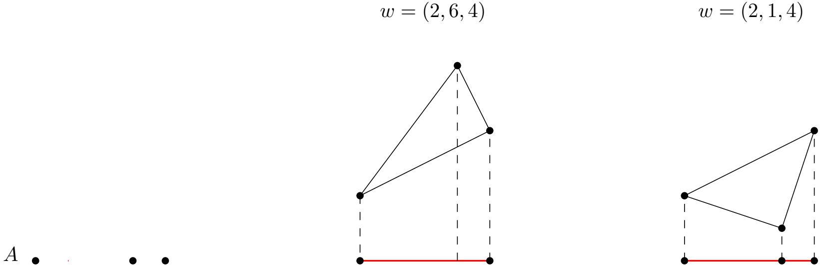
New Algorithm:

- Vertex oracle: given a direction vector compute a vertex of $N(R)$
- Output sensitive: computes only one triangulation of $\mathcal{A}$ per $N(R)$ vertex + one per $N(R)$ facet
- Computes projections of $N(R)$ or $\Sigma(\mathcal{A})$



A basic tool for the oracle:

Regular triangulations of $A \subset \mathbb{R}^d$ are obtained by projecting the lower (or upper) hull of A lifted to $\mathbb{R}^{d+1}$ via a generic lifting function $w \in (\mathbb{R}^{|\mathcal{A}|})^{\times}$.

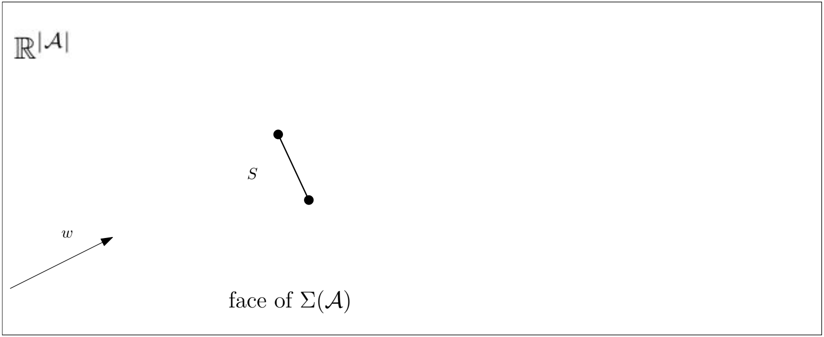


If w is not generic then we construct a regular subdivision.

The Vertex (Optimization) Oracle

Input: $\mathcal{A} \subset \mathbb{Z}^{2n}$, direction $w \in (\mathbb{R}^{|\mathcal{A}|})^\times$
Output: vertex $\in N(R)$, extremal wrt w

1. use w as a lifting to construct regular subdivision S of $\mathcal{A}$



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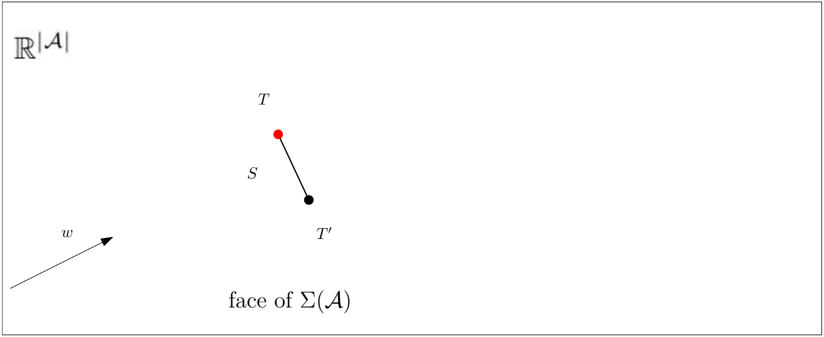
1. use w as a lifting to construct regular subdivision S of $\mathcal{A}$
2. refine S into triangulation T of $\mathcal{A}$



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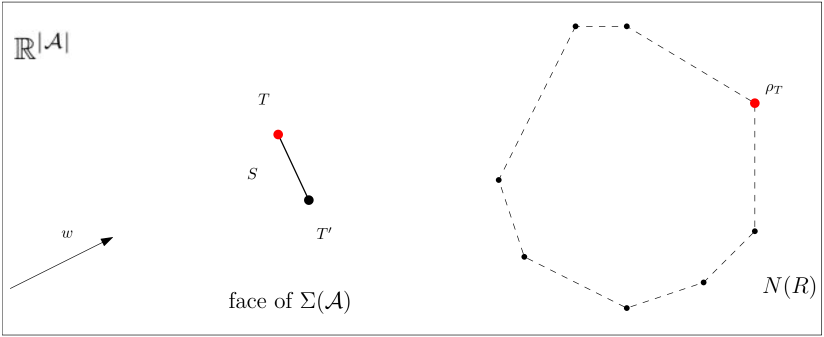
3. return $\rho_T \in \mathbb{N}^{|\mathcal{A}|}$



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Lemma

Oracle's output is

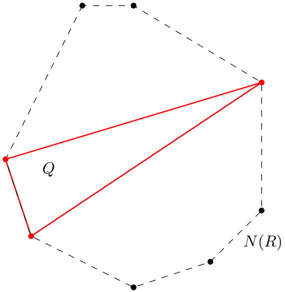
- *always a vertex of the target polytope,*
- *extremal wrt w.*

Incremental Algorithm

Input: $\mathcal{A}$

Output: H-rep. Q_H, V-rep. Q_V of $Q = N(R)$

1. initialization step



initialization:

- $Q \subset N(R)$
- dim(Q)=dim($N(R)$)

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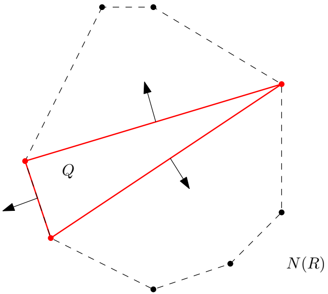

2 kinds of hyperplanes of Q_H :

- legal if it supports facet $\subset N(R)$
- illegal otherwise

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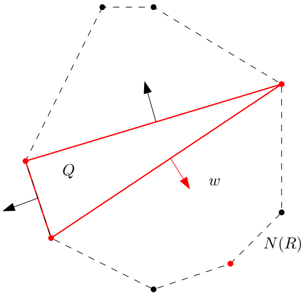

Extending an illegal facet

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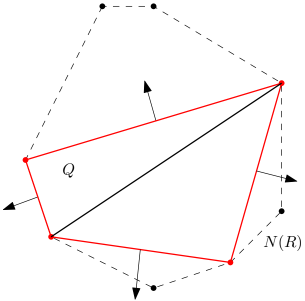

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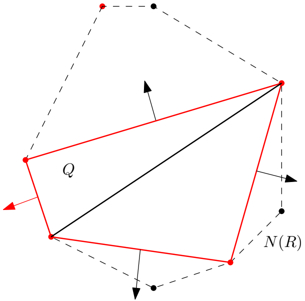

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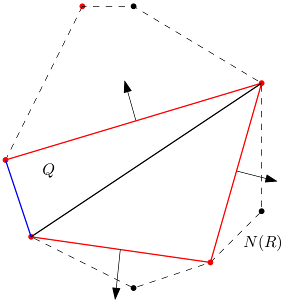

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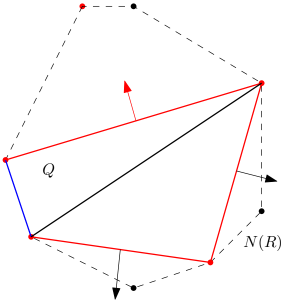

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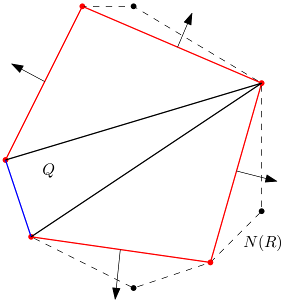

At any step, Q is an inner approximation ...

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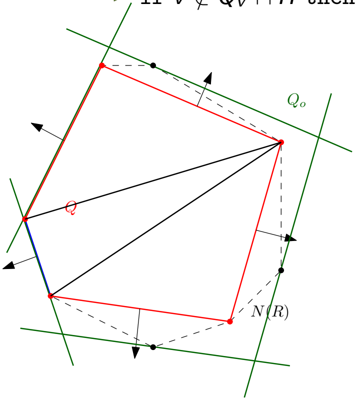

At any step, Q is an inner approximation ... from which we can compute an outer approximation Q_o.

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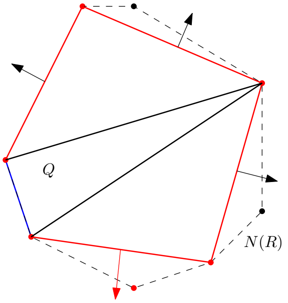

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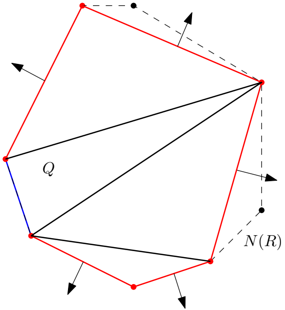

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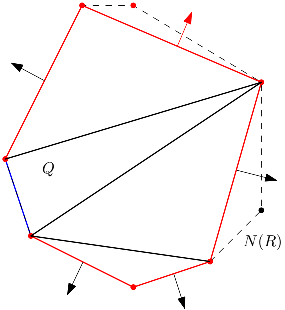

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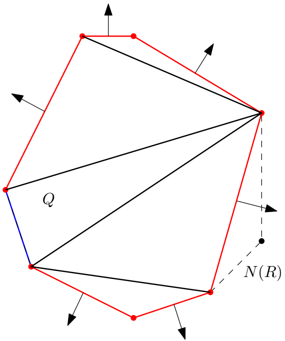

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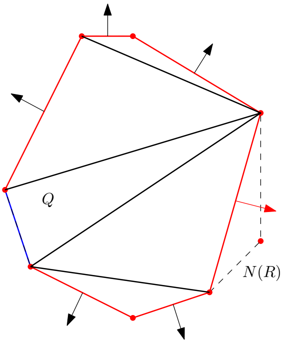

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Output: H-rep. Q_H, V-rep. Q_V of $Q = N(R)$

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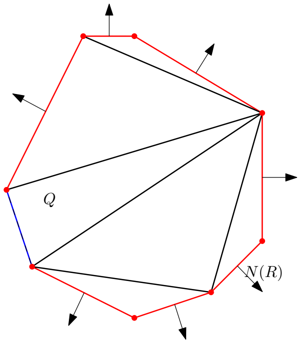

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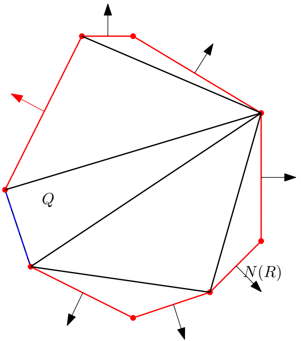

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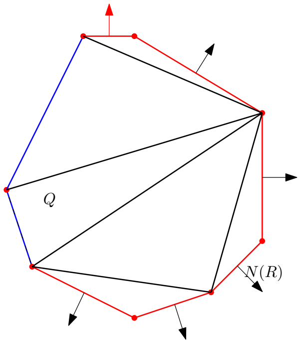

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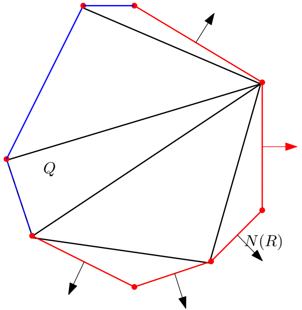

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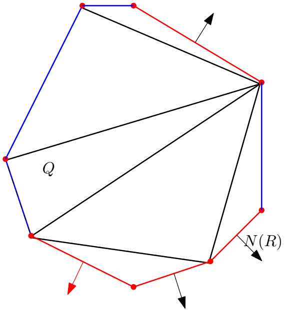

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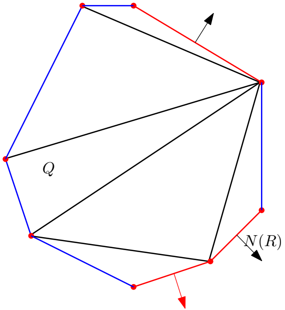

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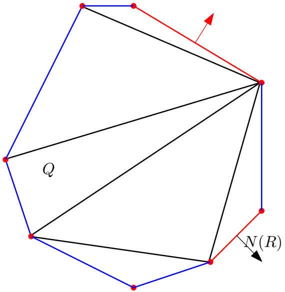

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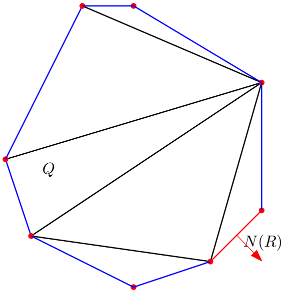

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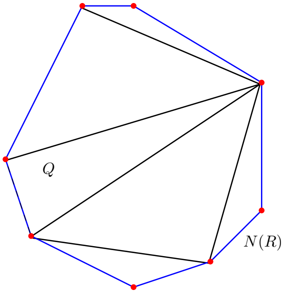


Complexity

Theorem

We compute the Vertex- and Halfspace-representations of $N(R)$, as well as a triangulation T of $N(R)$, in

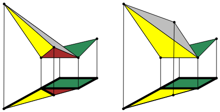
$$O^*(\, m^5 \, |vtx(N(R))| \cdot |T|^2),$$

where $m = \dim N(R)$, and $|T|$ the number of full-dim faces of T.

Elements of proof

- Computation is done in dimension $m = |\mathcal{A}| - 2n + 1$, $N(R) \subset \mathbb{R}^{|\mathcal{A}|}$.
- At most $\leq \mathrm{vtx}(N(R)) + \mathrm{fct}(N(R))$ oracle calls (Lem. 9).
- Beneath-and-Beyond algorithm for converting V-rep. to H-rep [Joswig '02].

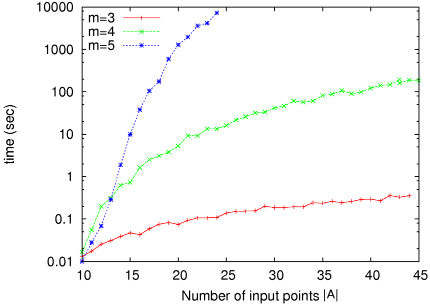
ResPol package

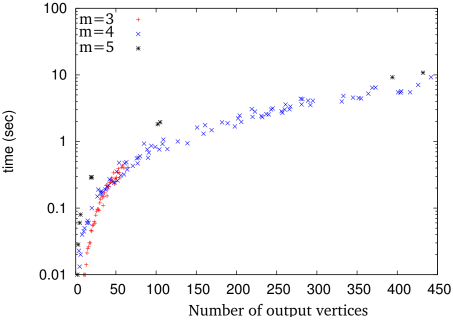


- C++
- towards high-dimensional CGAL
- `triangulation` [Boissonnat,Devillers,Hornus]
  `extreme_points_d` [Gärtner] (preprocessing step)
- Hashing of determinantal predicates: optimizing sequences of similar determinants
- `http://sourceforge.net/projects/respol`

Output-sensitivity

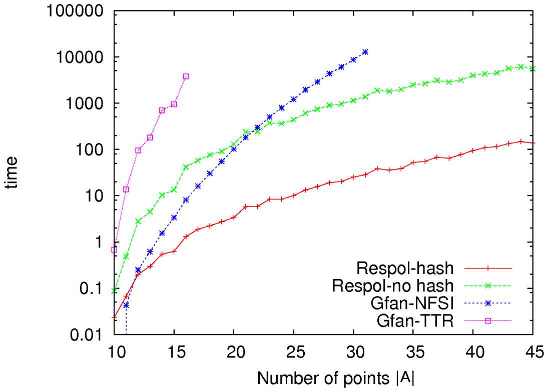
- oracle calls $\leq \mathrm{vtx}(N(R)) + \mathrm{fct}(N(R))$
- output vertices bound polynomially the output triangulation size
- subexponential runtime wrt to input points (L), output vertices (R)





Hashing and Gfan

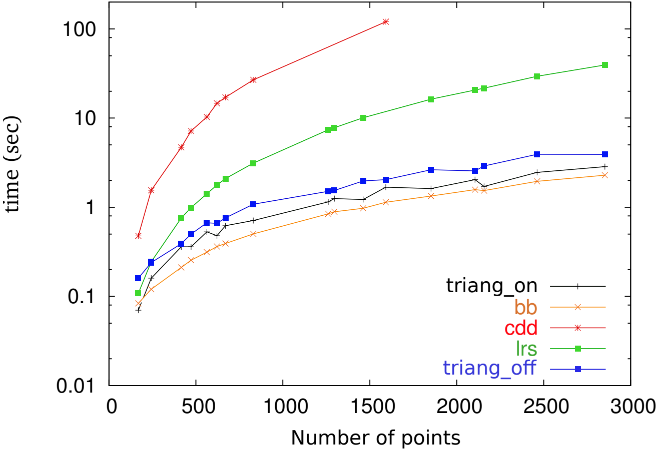
- *hashing determinants* speeds $\leq$ 10-100x when $dim(N(R)) = 3, 4$
- faster than `Gfan` [Yu-Jensen'11] for $dimN(R) \leq 6$, else competitive



$dim(N(R)) = 4$:

Computing the convex hull of N(R)

- `triangulation`, polymake beneath-beyond (`bb`), `cdd`, `lrs`



$dim(N(R)) = 4$

f-vectors of 4-dimensional N(R)

(6, 15, 18, 9)
(8, 20, 21, 9)
(9, 22, 21, 8)
.
.
.
(17, 48, 45, 14)
(17, 48, 46, 15)
(17, 48, 47, 16)
(17, 49, 47, 15)
(17, 49, 48, 16)
(17, 49, 49, 17)
(17, 50, 50, 17)
(18, 51, 48, 15)
(18, 51, 49, 16)
(18, 52, 50, 16)
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(19, 56, 54, 17)
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(19, 57, 57, 19)
(20, 58, 54, 16)
(20, 59, 57, 18)
(20, 60, 60, 20)
(21, 62, 60, 19)
(21, 63, 63, 21)
(22, 66, 66, 22)

Open

Almost symmetric f-vector?

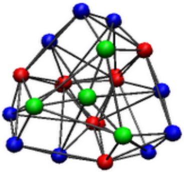
Ongoing and future work

- Extension of hashing determinants to CH computations (with L.Peñaranda) (to appear in ESA'12)
- Combinatorial characterization of 4-dimensional resultant polytopes (with I.Z.Emiris, A.Dickenstein)
- Computation of discriminant polytopes (with I.Z.Emiris, A.Dickenstein)
- Membership oracles from vertex (optimization) oracles (with B.Gärtner)

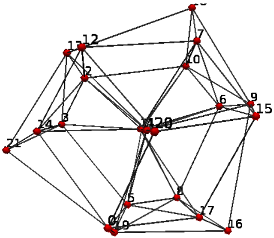
References

- The paper: "An output-sensitive algorithm for computing projections of resultant polytopes." in SoCG'12
- The code: `http://respol.sourceforge.net`

The end...



(figure courtesy of M.Joswig)



Facet and vertex graph of the largest 4-dimensional resultant polytope

Thank You !