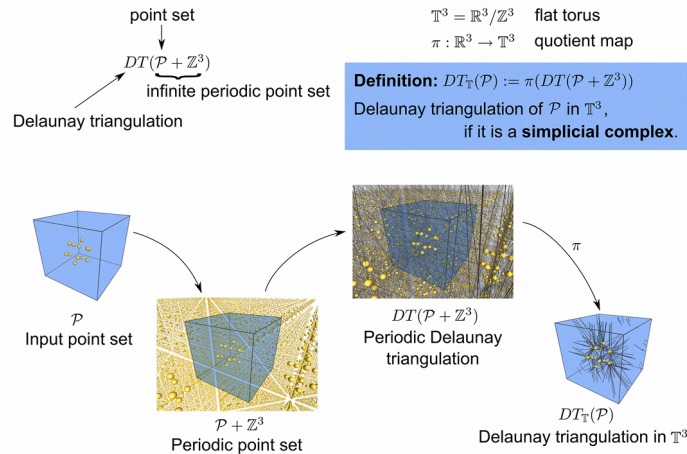
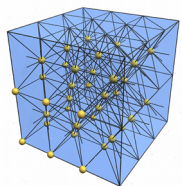


Delaunay triangulations in \mathbb{T}^3 [CT09]



There is a set \mathcal{Q} of 36 points such that $DT_{\mathbb{T}^3}(\mathcal{Q} \cup \mathcal{P})$ is a **simplicial complex** for any point set \mathcal{P} .



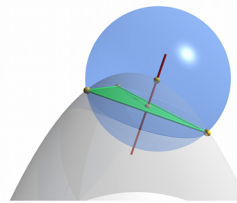
Algorithm

1. **Initialize** with point set \mathcal{Q} of 36 points
2. **Insert** points one by one into the Delaunay triangulation

CGAL Periodic Triangulations [CT10]

- generic, robust, fully dynamic
- Voronoi functionality
- randomized worst-case optimal $O(n^2)$
- efficient in practice

Meshing surfaces [BO05]



Surface
 Boundary facet
 Dual Voronoi edge
 Surface Delaunay ball

Refinement criteria

Aspect criterion

- lower bound on minimum angle of **boundary facet**

Uniform size criterion

- upper bound on radius of **surface Delaunay ball**

Curvature criterion

- upper bound on distance between center of a **boundary facet** and center of its **surface Delaunay ball**

Given a C^2 -continuous surface S without boundary, the algorithm generates a triangulated surface with

- the same topological type as S
- close to S for the Hausdorff distance
- good approximation for normals, areas, and curvature



Algorithm

bad facets: boundary facets that do not meet the refinement criteria

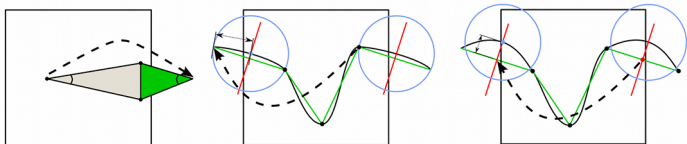
1. **Initialization step**
Find points on surface and construct their Delaunay triangulation.
2. **Refinement step**
For each **bad facet**, insert the center of its surface Delaunay ball into the Delaunay triangulation.

Triangulation

CGAL Surface Mesher [RY10]

Criteria

Periodic refinement criteria



Aspect criterion

translate the vertex to compute the angle

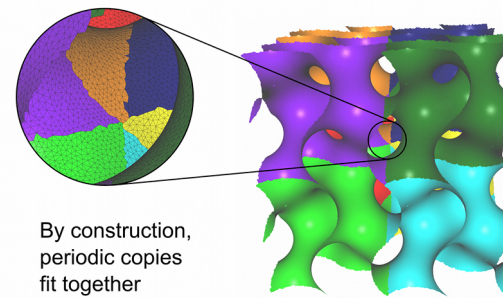
Uniform size criterion

translate the vertex to compute the **sphere radius**

Curvature criterion

translate the **facet center** to compute the distance to the surface

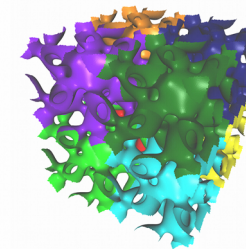
Result



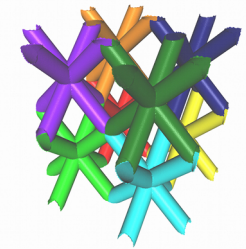
By construction, periodic copies fit together

Examples

Angular bound 30°, Radius bound 0.01, Distance bound 0.01:

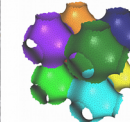


Lidinoid

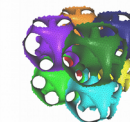


Surface used in bone scaffolding
 (Courtesy of M. Moesen, K. U. Leuven)

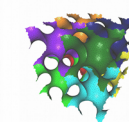
Angular bound 30°, Radius bound 0.03, Distance bound 0.03:



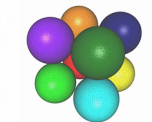
Schwarz p



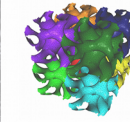
Double p



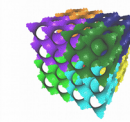
Gyroid



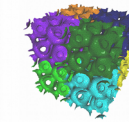
Sphere



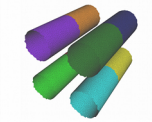
Gyroid



Diamond



D prime



Cylinder

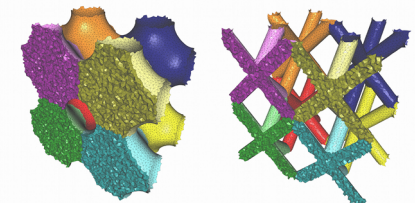
Current work: Volume meshes

Facet criteria

Angular bound: 30°
 Radius bound: 0.03
 Distance bound: 0.03

Cell criteria

Radius-edge bound: 2
 Radius bound: 0.03



(Joint work with Mikhail Bogdanov)

References

- [BO05] Jean-Daniel Boissonat and Steve Oudot. Provably good surface sampling and approximation. In *Graphical Models*, 67405-431, 2005.
- [CT09] Manuel Caroli and Monique Teillaud. Computing 3D Periodic Triangulations. In *Proceedings of the 17th European Symposium on Algorithms*, LNCS 5757, pages 37-48, 2009.
- [CT10] Manuel Caroli and Monique Teillaud. 3D Periodic Triangulations. In *CGAL 3.6 User and Reference Manual*, 2010.
- [RY10] Laurent Rineau and Mariette Yvinec. 3D Surface Mesh Generation. In *CGAL 3.6 User and Reference Manual*, 2010.