Meshing Periodic Surfaces in CGAL

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advisor: Monique Teillaud

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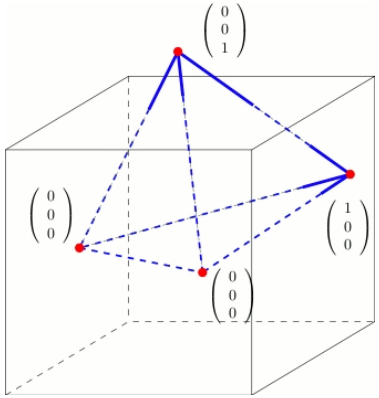
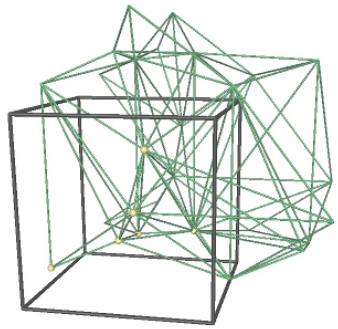
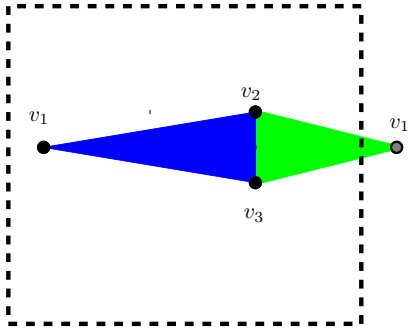
Outline





Periodic Triangulations



Periodic Space

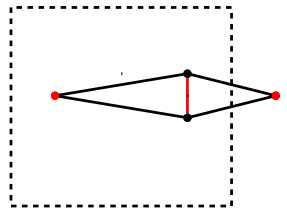
- $\mathbb{T}^3_c := \mathbb{R}^3/\mathbb{Z}^3$

Definitions

- domain: cube contains exactly one representative of each element
- compute in original domain (one contains the origin)
- offset (o_x, o_y, o_z): the number of periods far from the original domain

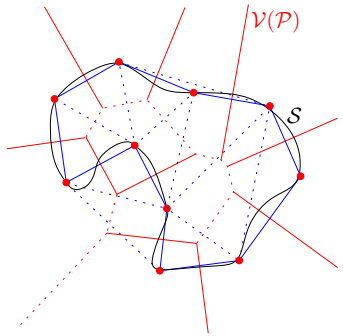
Valid Triangulations

- problem: not all point sets have a triangulation in $\mathbb{T}^3_c$
- solutions [CT09]:
  - compute in 27-sheeted covering space
  - insert 36 dummy points at the beginning



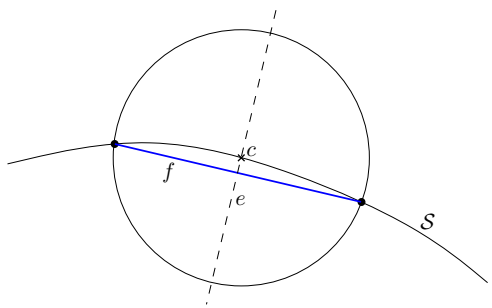
Surface Meshing - Definitions

Restricted Delaunay triangulation (rDt) 2D complex in 3D Delaunay triangulation of facets whose dual Voronoi segments intersect surface



Surface Delaunay ball

ball circumscribing facet with center an intersection of dual Voronoi edge of facet with the surface

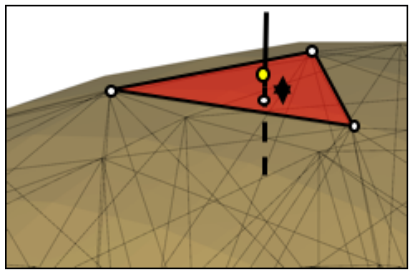


Boundary Facets

facets of restricted Delaunay triangulation

Surface Meshing - Refinement Criteria

Criteria



- angular bound
  minimum angle
- radius bound
  radius of the surface delaunay ball
- distance bound
  center of a boundary facet and center of its surface delaunay ball

Bad Facets: boundary facets not meeting criteria

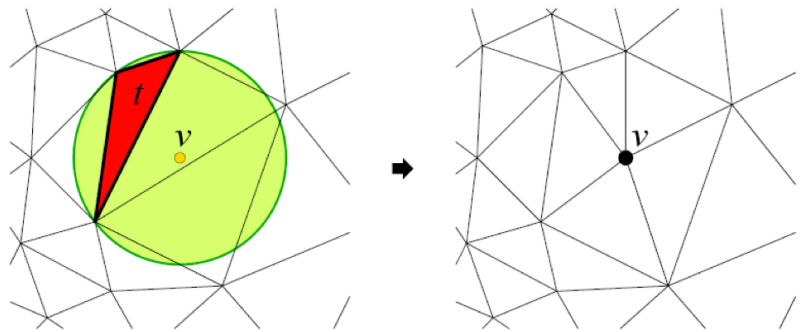
Surface Meshing - The Algorithm

Initialization Step

- find points on surface (ray shooting, dichotomic search)
- insert points to triangulation
- construct rDt and queue for bad facets

Refinement Step

- for each bad facet:
  - insert the center of its surface delaunay ball
  - update rDt and bad facets' queue

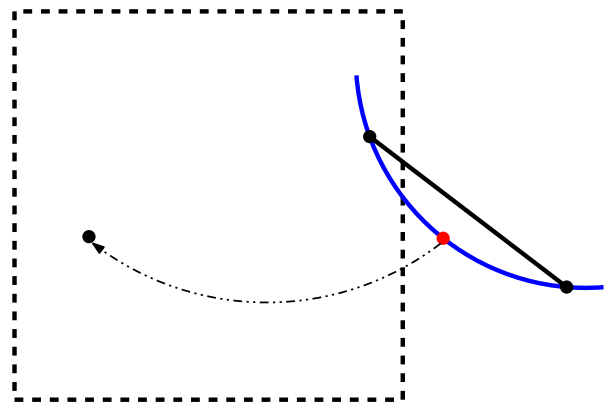


Periodic Surface Meshing - Algorithm

CGAL 3D Surface Mesher with 3D Periodic Delaunay Triangulations

Modifications

- Point insertion
  (Initialization step, Refinement step)
  translate points inside the domain (use offsets)

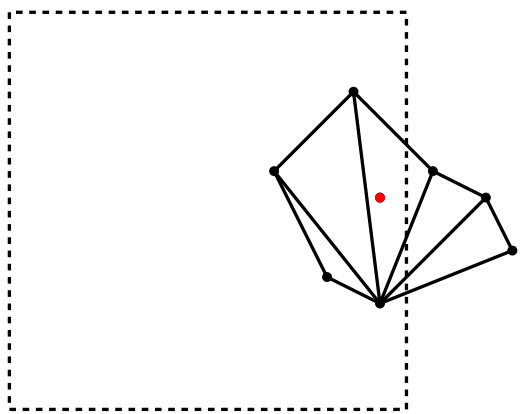


Periodic Surface Meshing - Algorithm

CGAL 3D Surface Mesher with 3D Periodic Delaunay Triangulations

Modifications

- Point insertion
  (Initialization step, Refinement step)
  translate points inside the domain (use offsets)
- Star approach
  keep track of offsets during insertion



Periodic Surface Meshing - Algorithm

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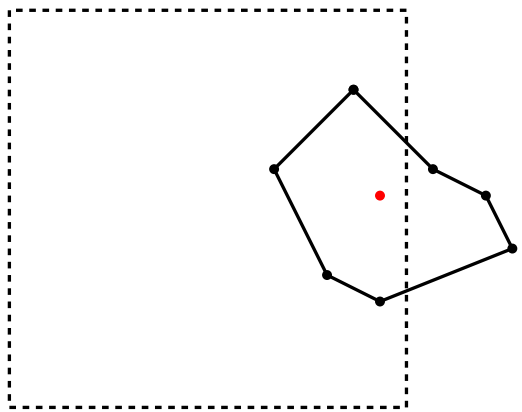

Periodic Surface Meshing - Algorithm

CGAL 3D Surface Mesher with 3D Periodic Delaunay Triangulations

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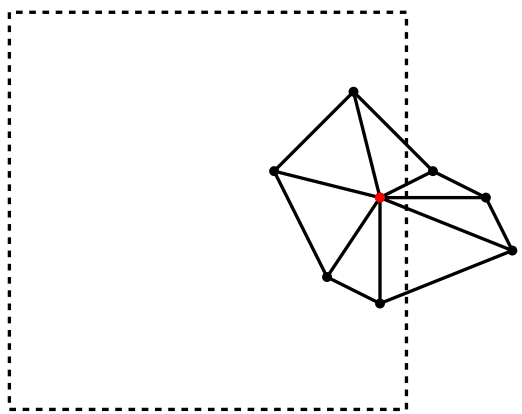


Periodic Surface Meshing - Algorithm

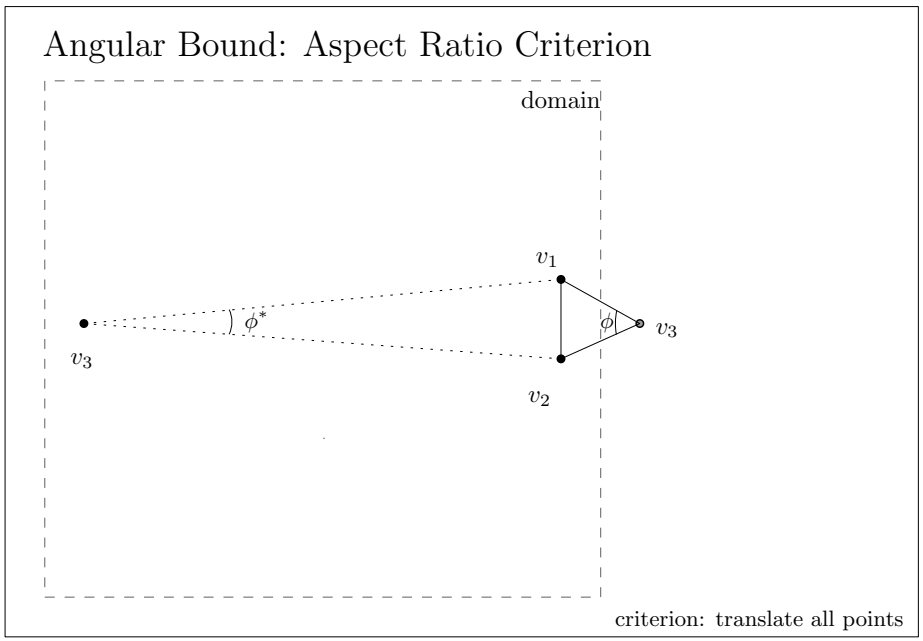
CGAL 3D Surface Mesher with 3D Periodic Delaunay Triangulations

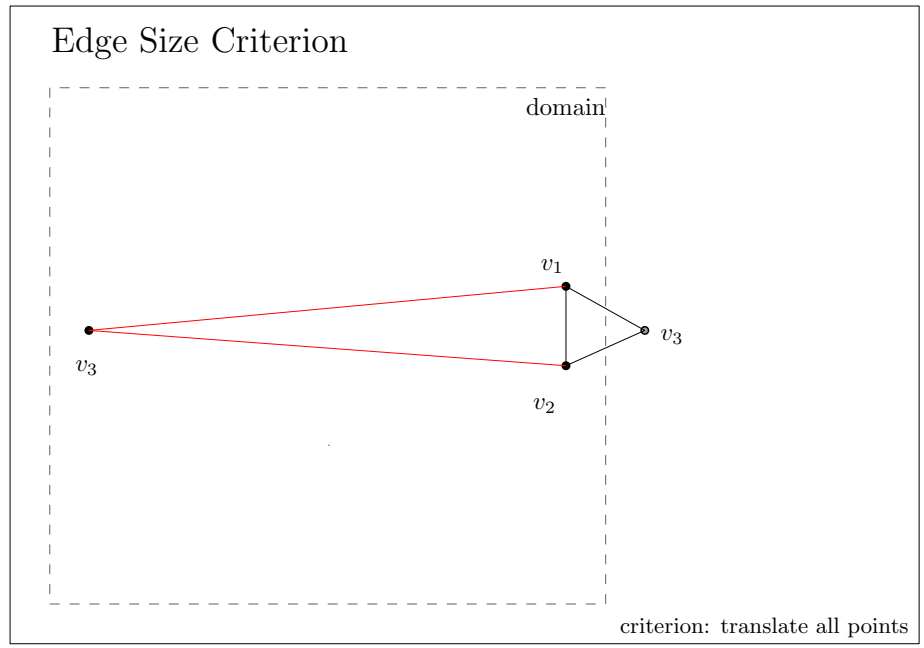
Modifications

- Point insertion
  (Initialization step, Refinement step)
  translate points inside the domain (use offsets)
- Star approach
  keep track of offsets during insertion
- Refinement Criteria
  algorithm might compute the wrong quantity

Periodic Surface Meshing - Criteria (1)

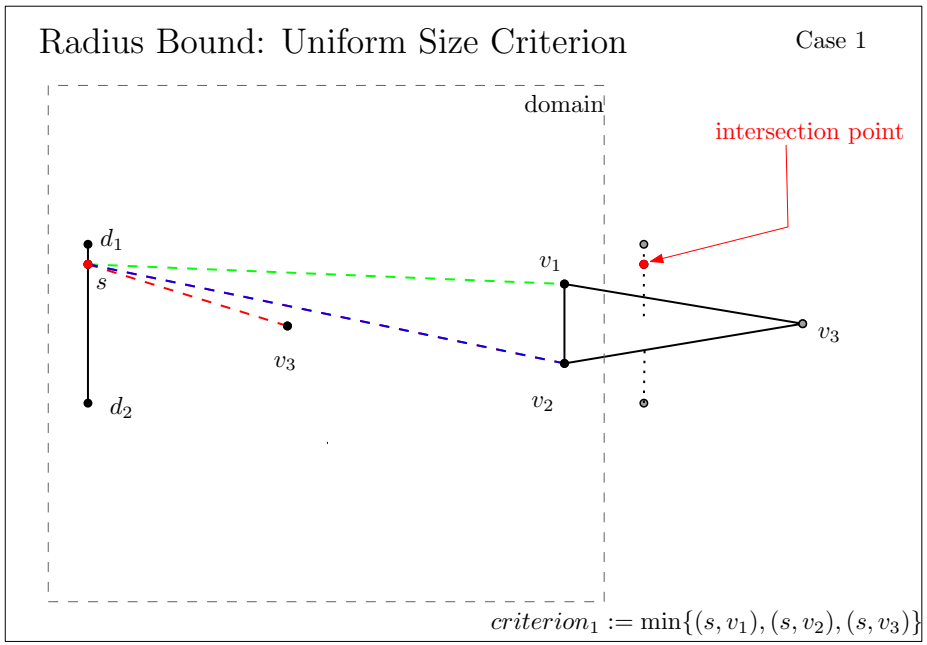
Aspect Ratio Criterion (Angular Bound) - Edge Length Criterion

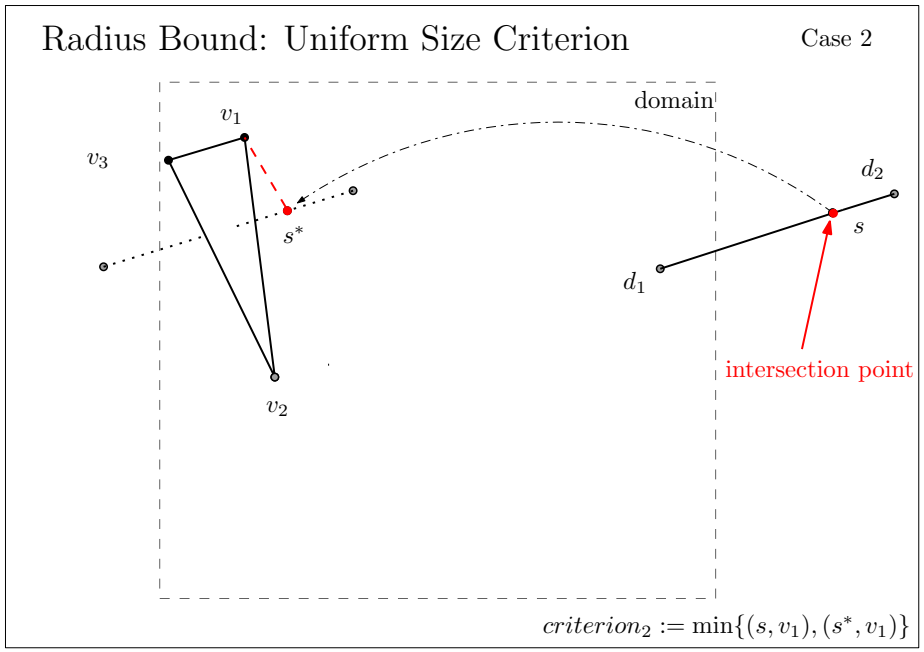




Periodic Surface Meshing - Criteria (2)

Uniform Size Criterion (Radius Bound)

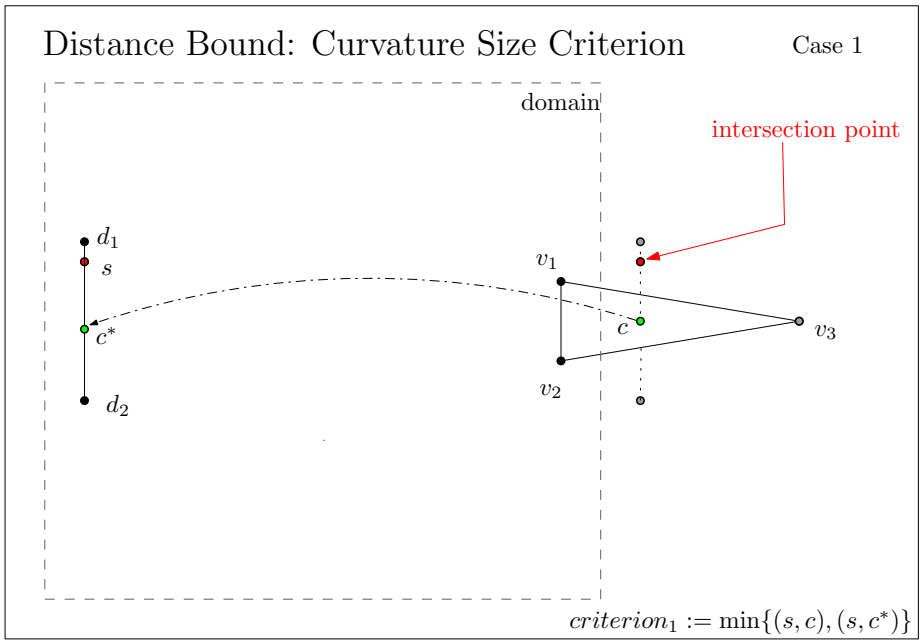


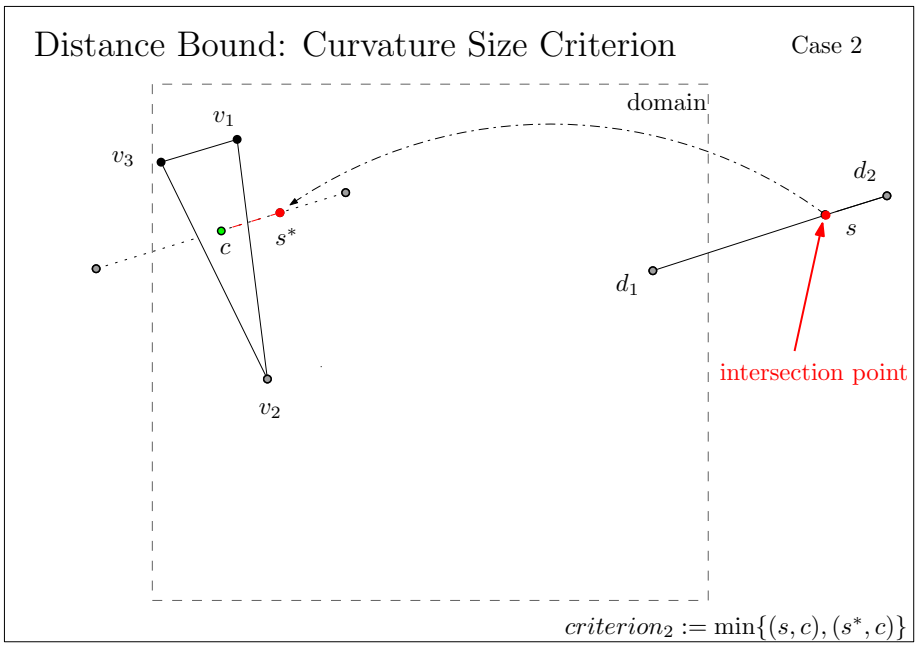


criterion: $\min\{(s, v_1), (s, v_2), (s, v_3), (s^*, v_1)\}$

Periodic Surface Meshing - Criteria (3)

Curvature Size Criterion (Distance Bound)





criterion: $\min\{(s,c),(s,c^*),(s^*,c)\}$

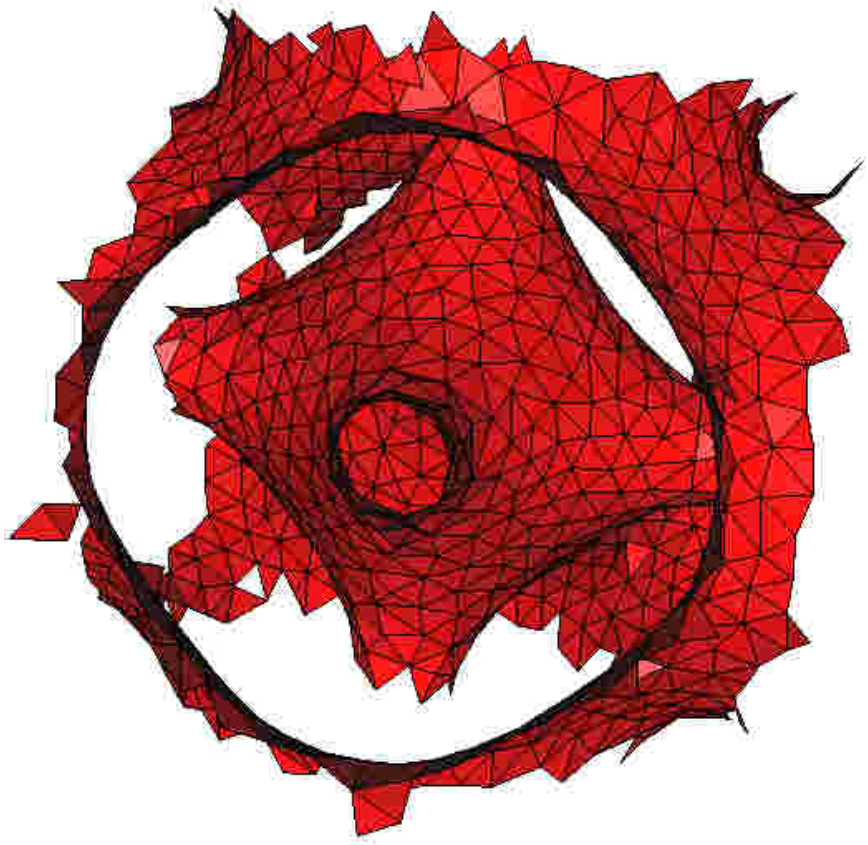
Examples

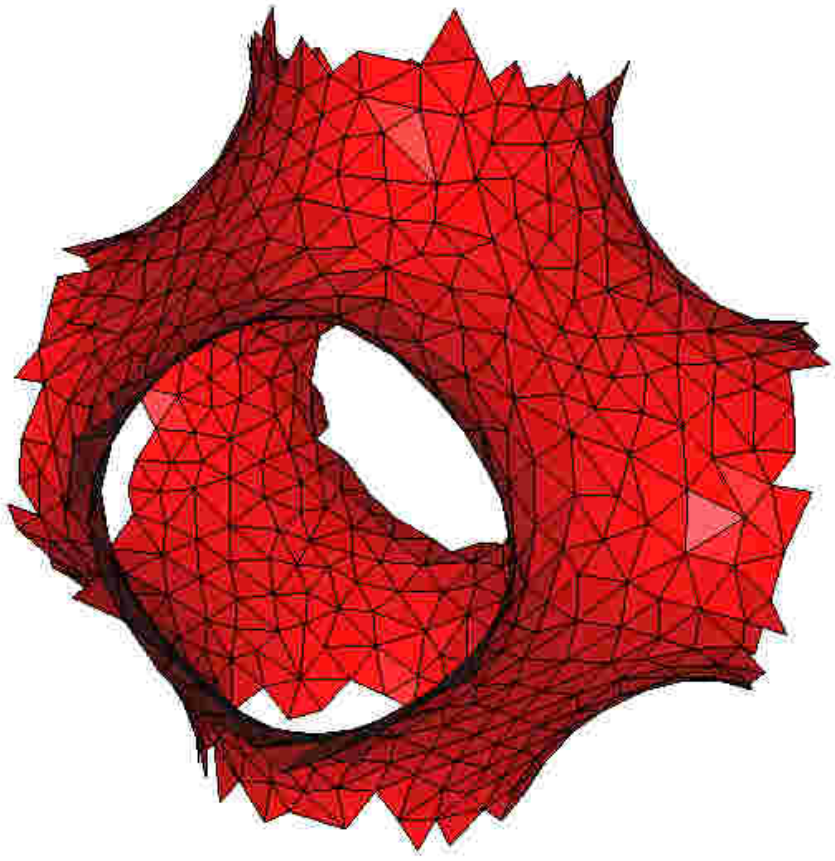
double p

$$0 = 0.5 \cdot (cosx \cdot cosy + cosy \cdot cosz + cosz \cdot cosx) + 0.2 \cdot (cos2x + cos2y + cos2z)$$

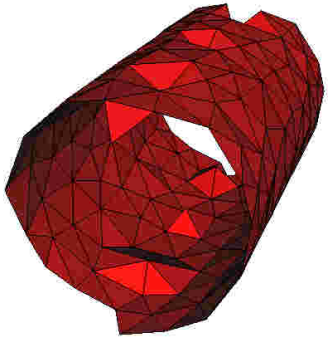
schwarz p

$$0 = cosx + cosy + cosz$$

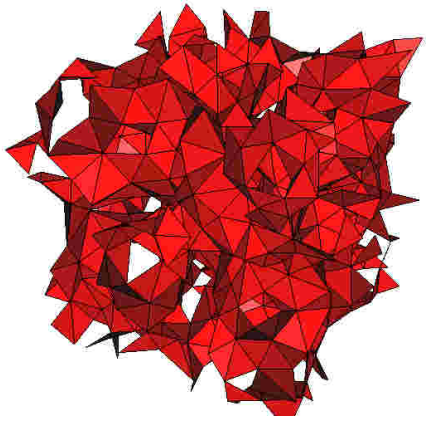




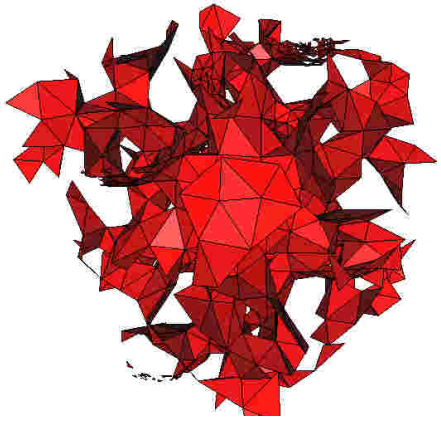
criteria: $angular = 30$ $radius = 0.1$ $distance = 0.1$



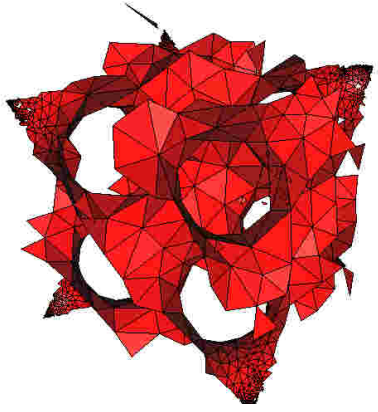
cylinder



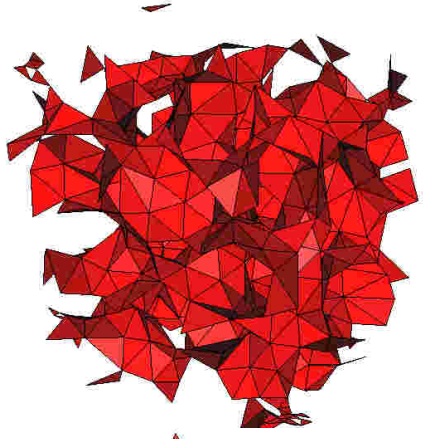
D prime



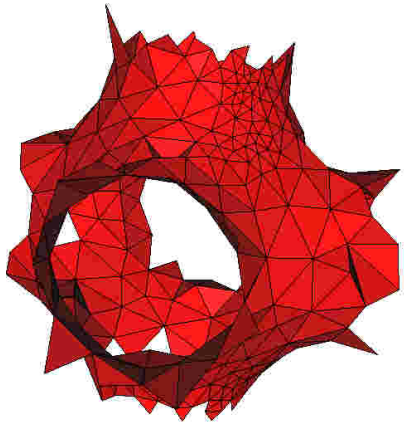
lidinoid



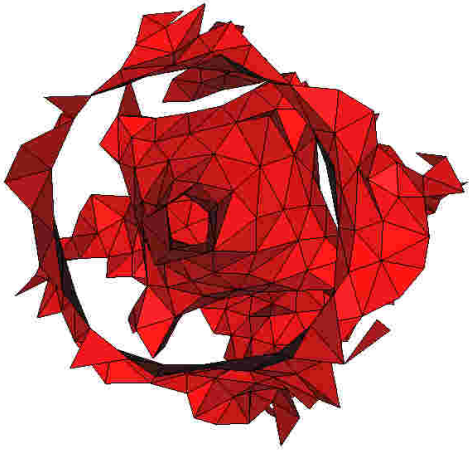
diamond



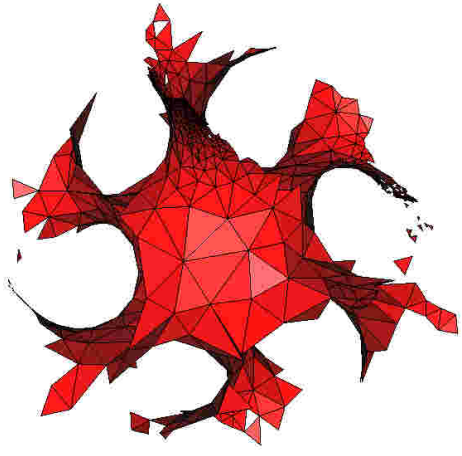
G prime



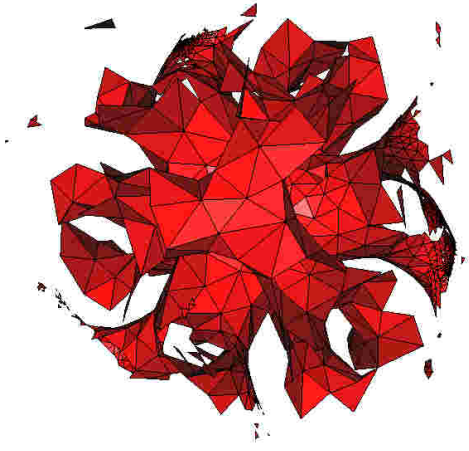
schwarz p



double p

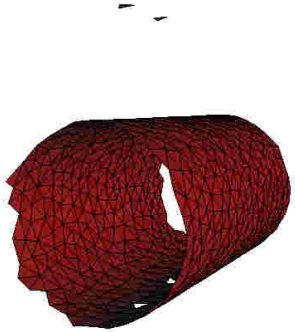


gyroid

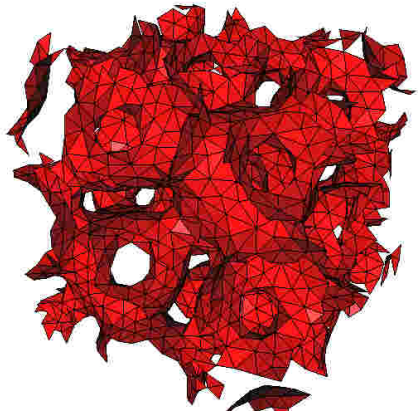


split p

criteria: $angular = 30$ $radius = 0.05$ $distance = 0.05$



cylinder



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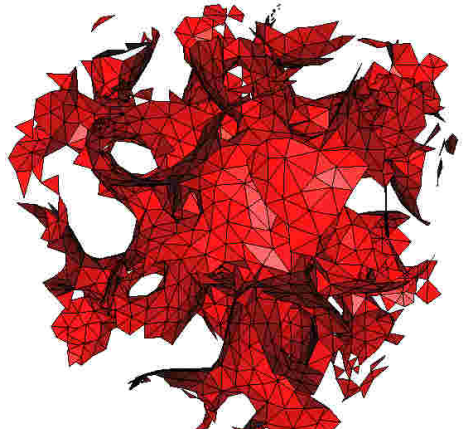
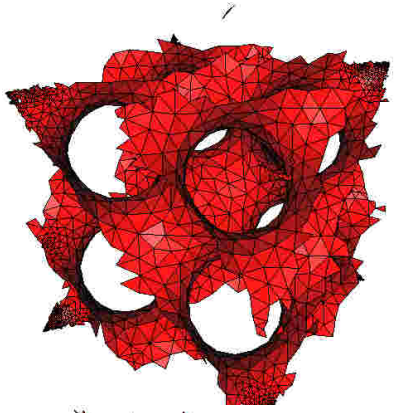


lidinoid



diamond

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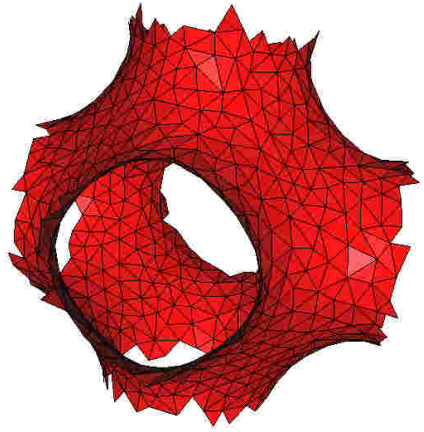
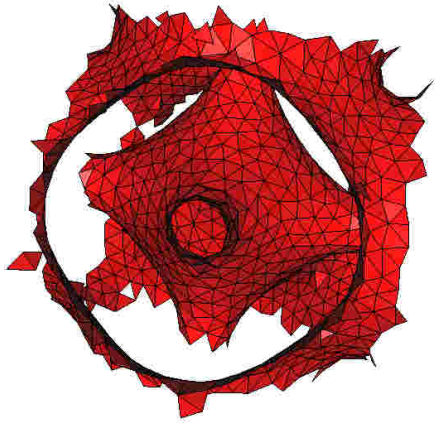
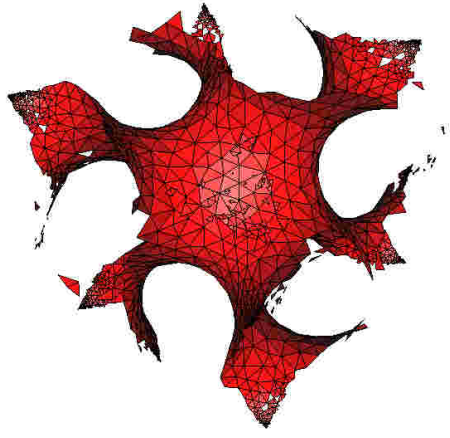


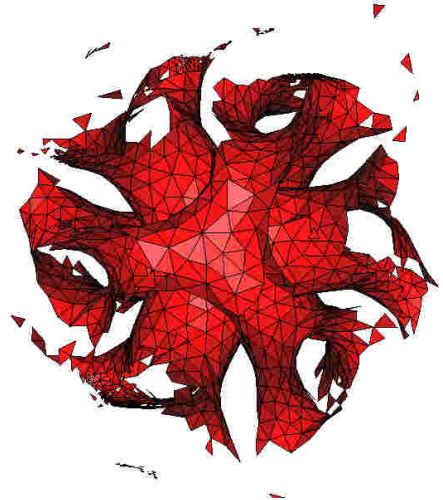
schwarz p



double p

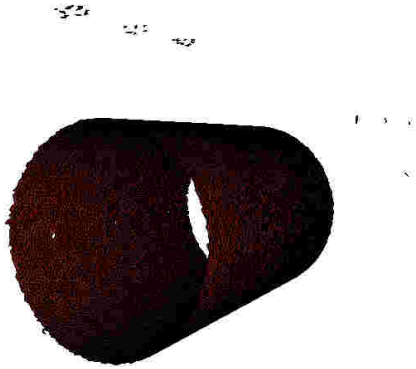


gyroid



split p

criteria: $angular = 30$ $radius = 0.01$ $distance = 0.01$



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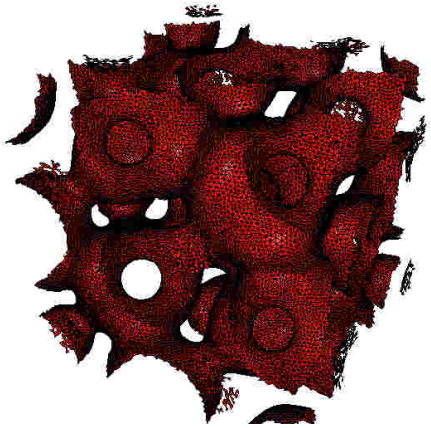

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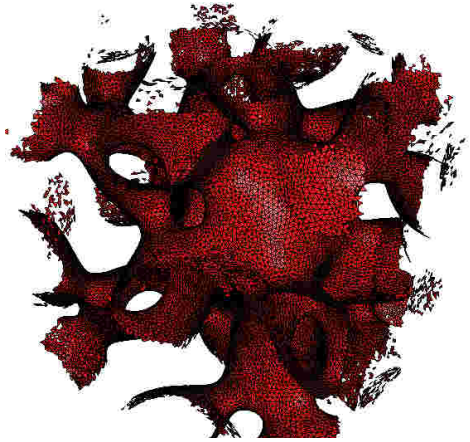

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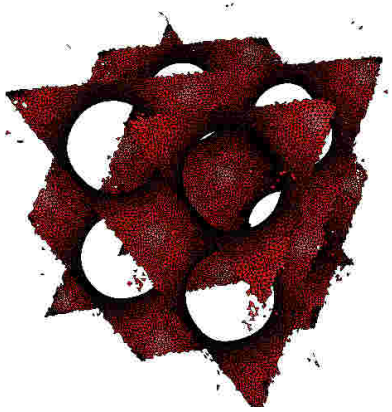
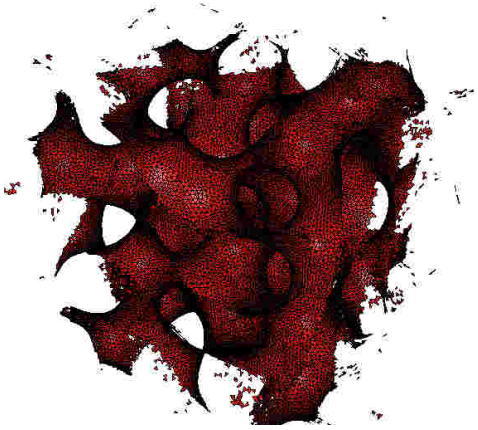
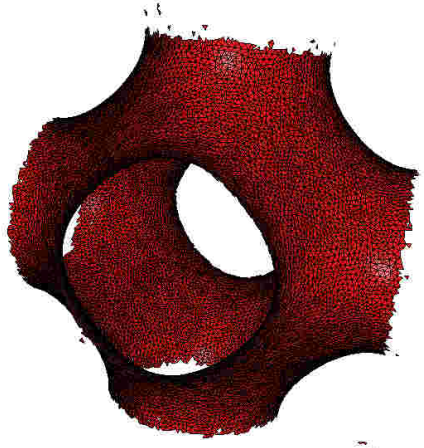


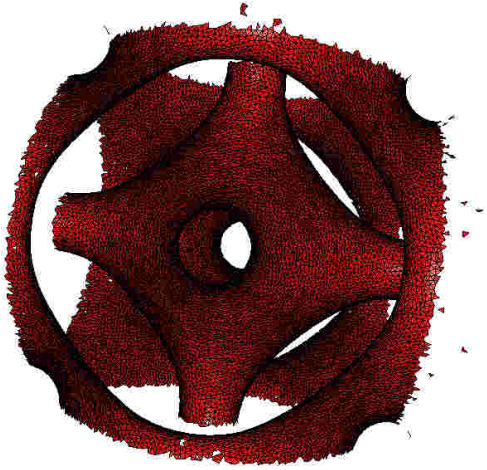
diamond



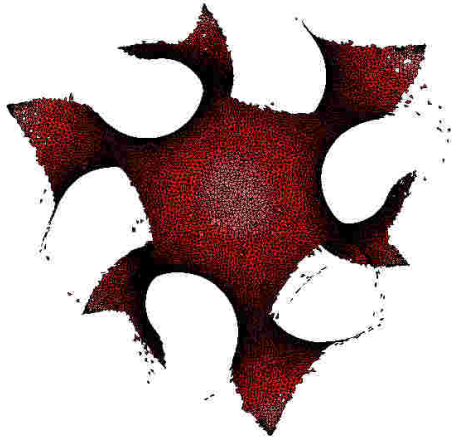
G prime



schwarz p



double p



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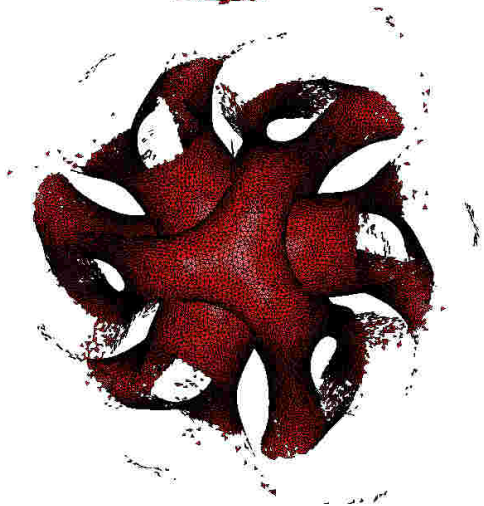
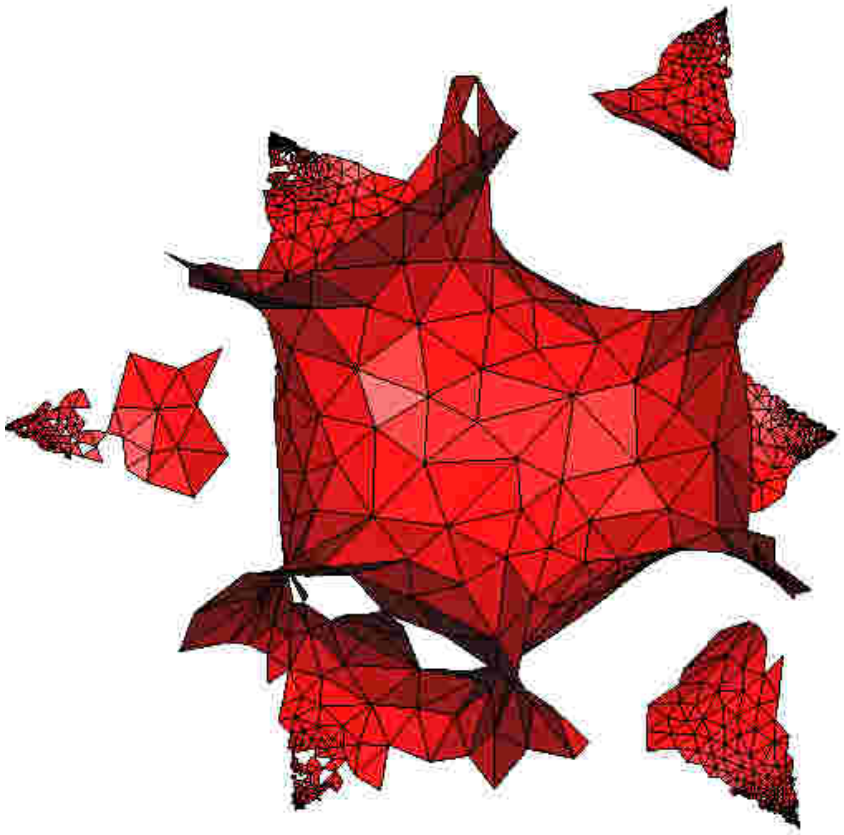


split p

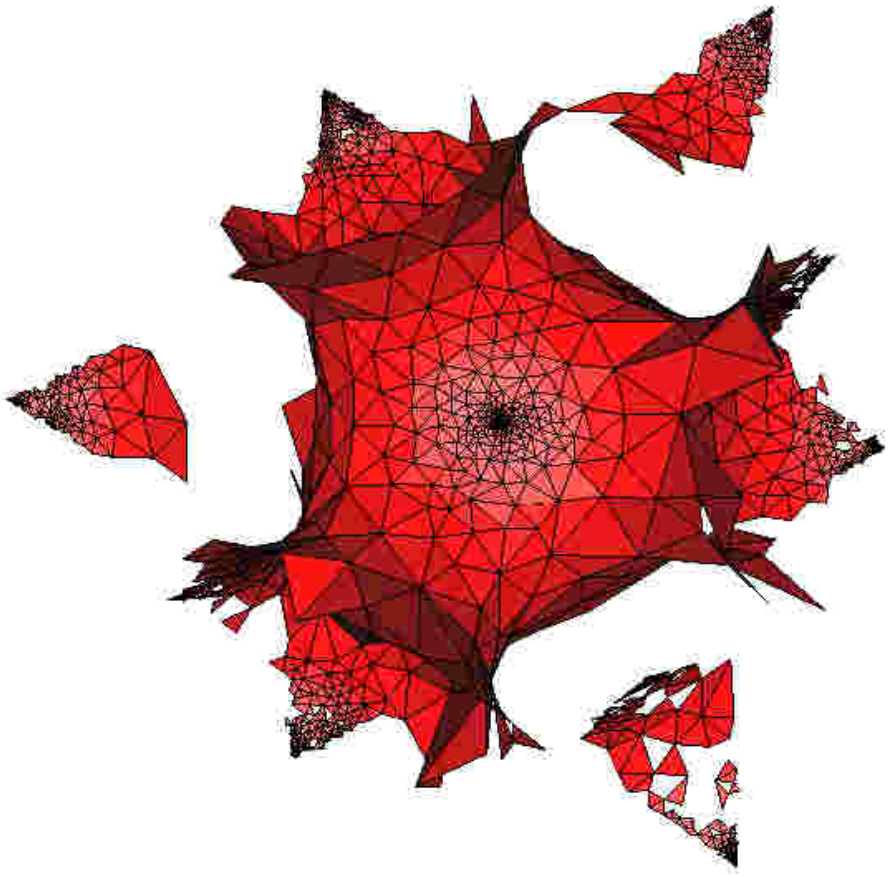
Bad Examples

schwarz p: $0 = cos(2\pi x + \frac{\pi}{2}) + cos(2\pi y + \frac{\pi}{2}) + cos(2\pi z + \frac{\pi}{2})$

$angular = 30$ $radius = 0.1$ $distance = 0.1$



$angular = 30$ $radius = 0.05$ $distance = 0.05$



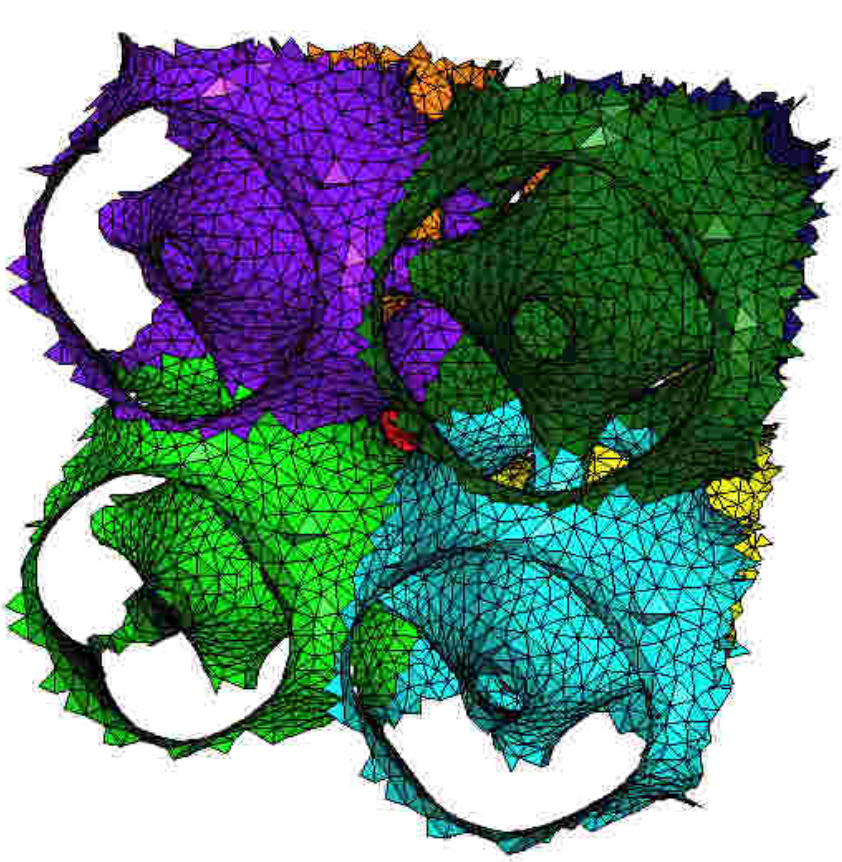
Examples

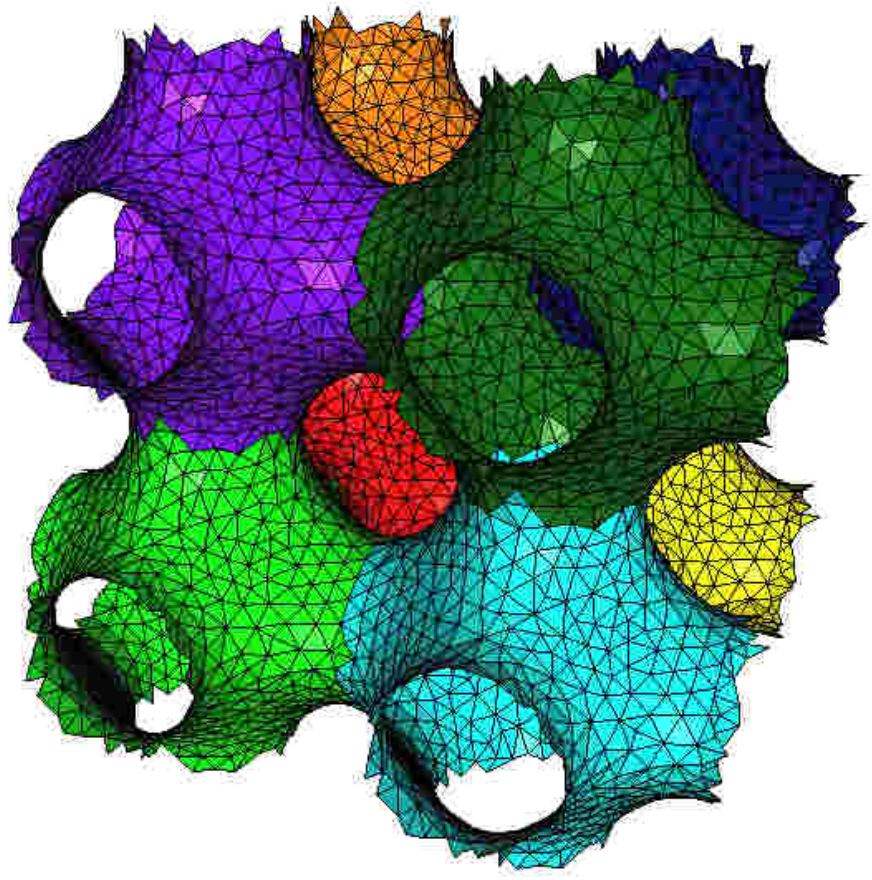
double p

$$0 = 0.5 \cdot (cosx \cdot cosy + cosy \cdot cosz + cosz \cdot cosx) + 0.2 \cdot (cos2x + cos2y + cos2z)$$

schwarz p

$$0 = cosx + cosy + cosz$$





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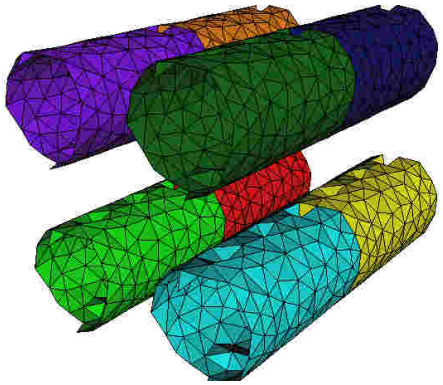
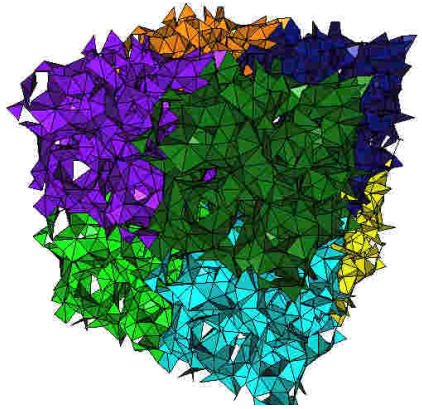
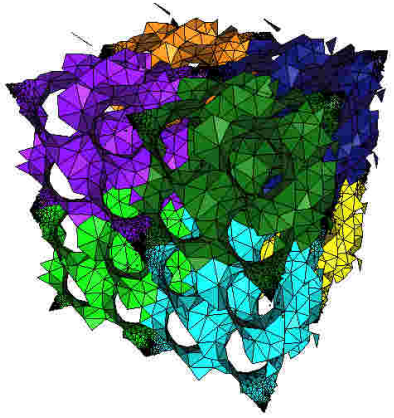


cylinder

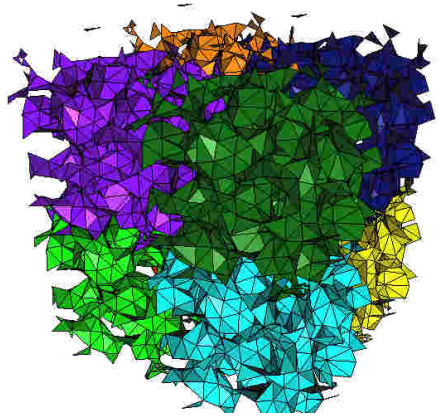


D prime

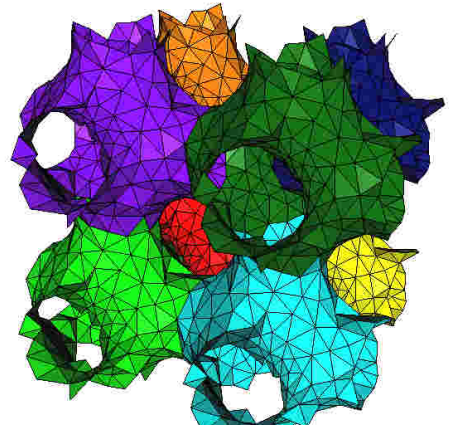
lidinoid



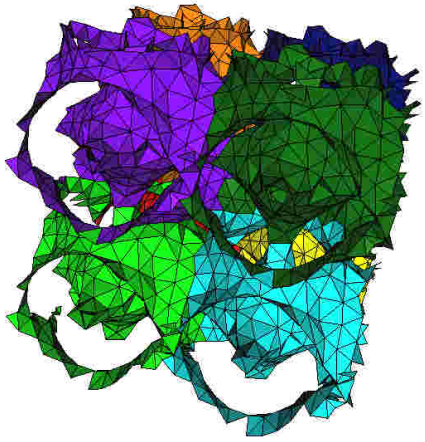
diamond



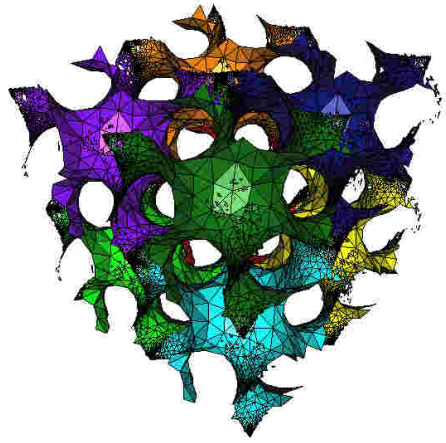
G prime



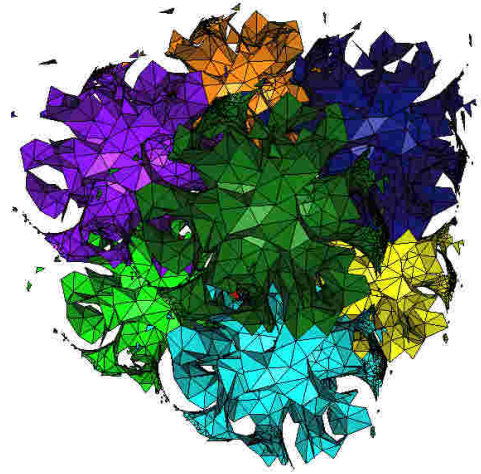
schwarz p



double p

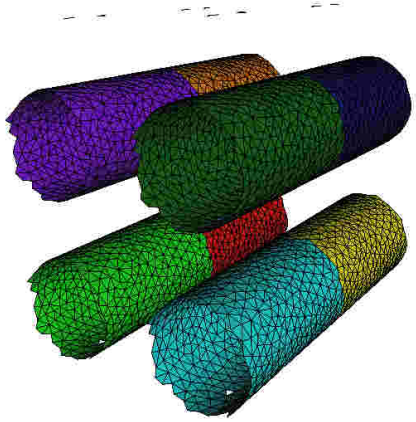


gyroid

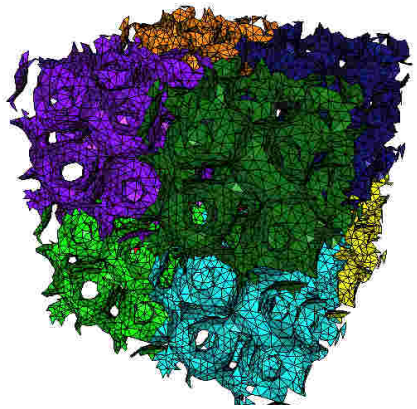


split p

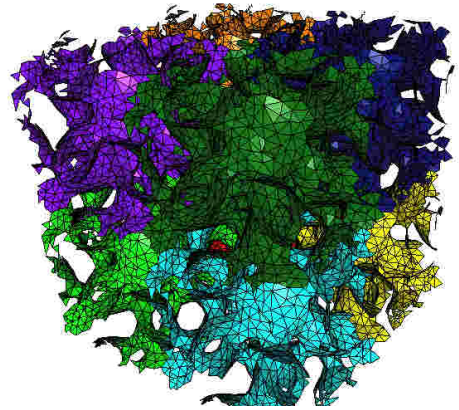
criteria: $angular = 30$ $radius = 0.05$ $distance = 0.05$



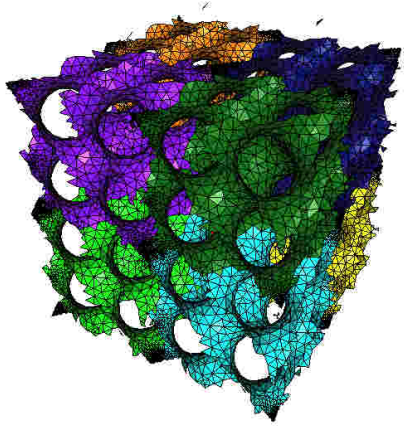
cylinder



D prime



lidinoid



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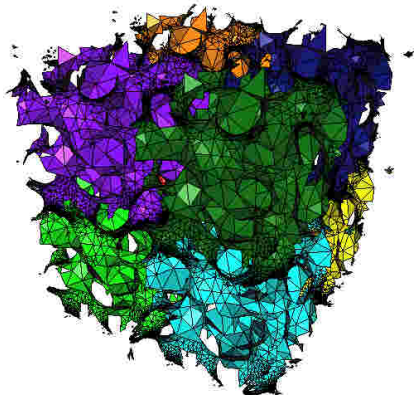
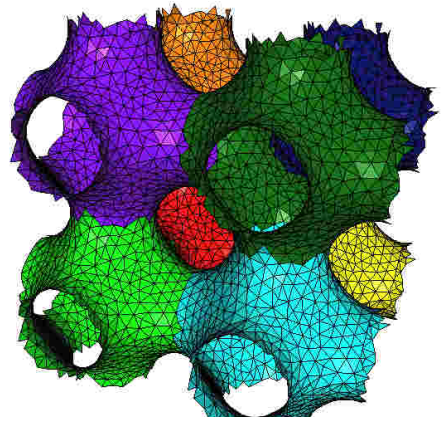
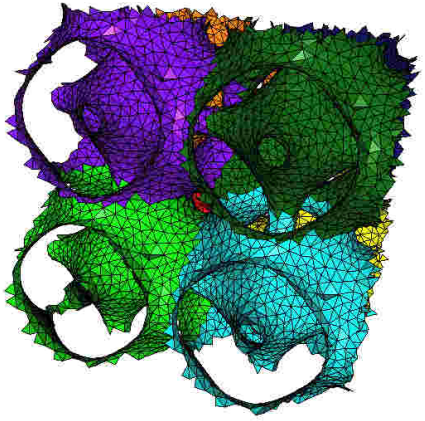


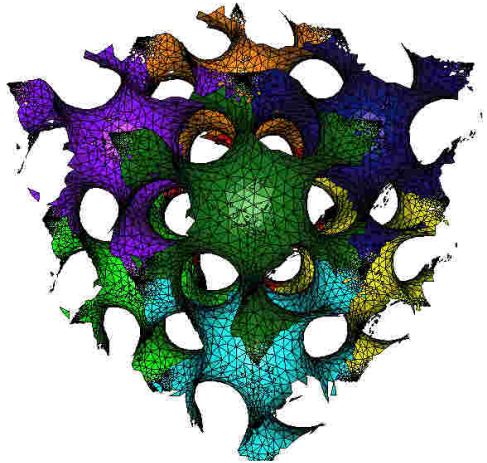
G prime



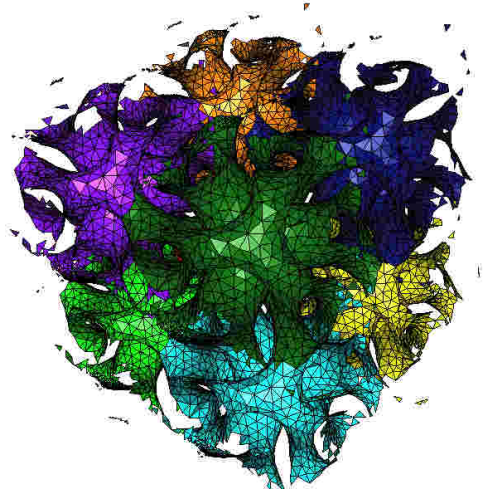
schwarz p



double p



gyroid



split p

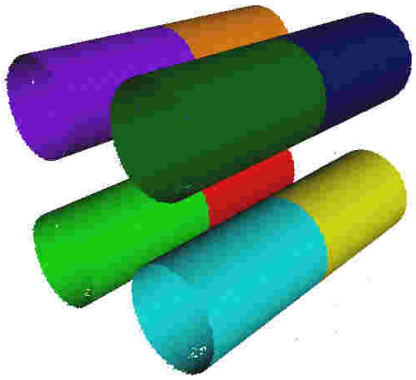
Conclusion - Future Work

Conclusions

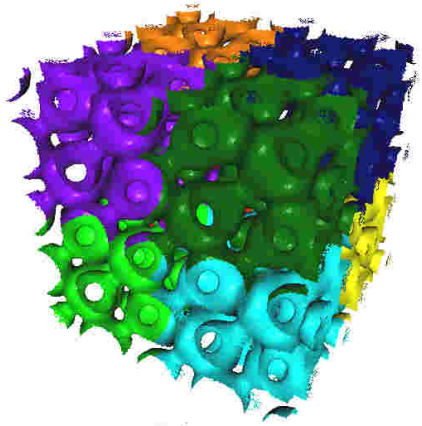
- The algorithm terminates for all of the examples
- The algorithm can mesh many periodic minimal surfaces

Future Work

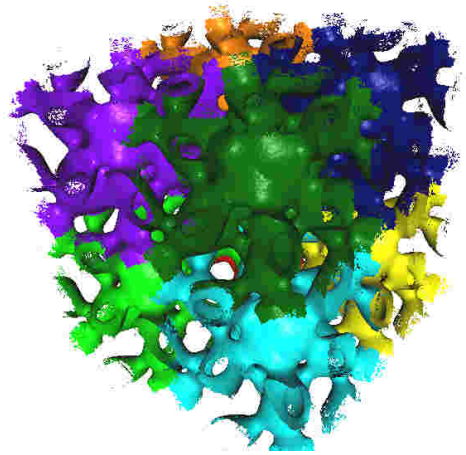
- Improve the refinement criteria in order to handle all cases
- Prove the correctness and termination for the new criteria



cylinder



D prime



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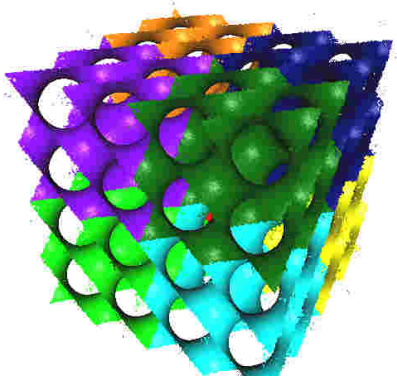
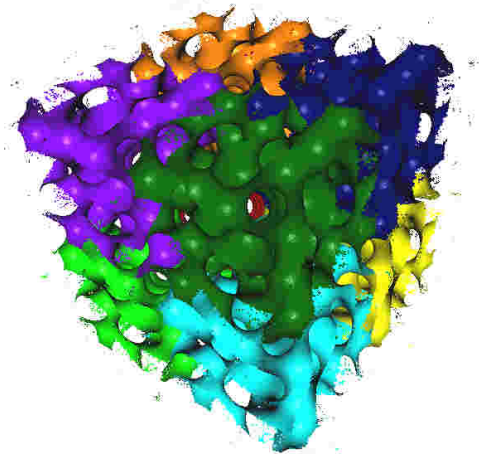
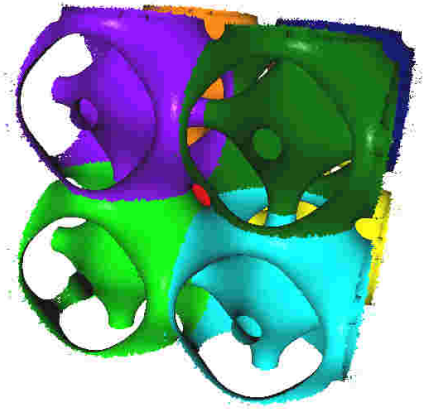


diamond

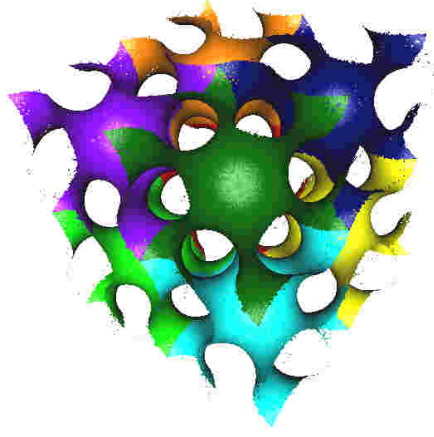


G prime

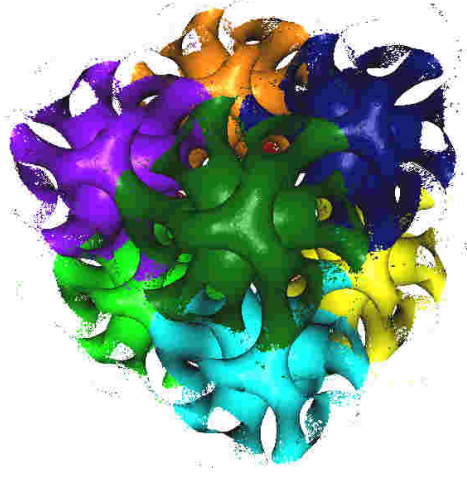
schwarz p



double p



gyroid



split p

Thank You for Your Attention!

Appendix

cylinder:

$$0 = x^2 + y^2$$

diamond:

$$0 = sinx \cdot siny \cdot sinz + sinx \cdot cosy \cdot cosz + cosx \cdot siny \cdot cosz + cosx \cdot cosy \cdot sinz$$

double p:

$$0 = 0.5 \cdot (cosx \cdot cosy + cosy \cdot cosz + cosz \cdot cosx) + 0.2 \cdot (cos2x + cos2y + cos2z)$$

D prime:

$$0 = 1 \cdot (sinx \cdot siny \cdot sinz) + 1 \cdot (cosx \cdot cosy \cdot cosz) - 1 \cdot (cos2x \cdot cos2y + cos2y \cdot cos2z + cos2z \cdot cos2x)) - 0.4$$

G prime:

$$0 = 5 \cdot (sin2x \cdot sinz \cdot cosy + sin2y \cdot sinx \cdot cosz + sin2z \cdot siny \cdot cosx) + 1 \cdot (cos2x \cdot cos2y + cos2y \cdot cos2z + cos2z \cdot cos2x)$$

Appendix

gyroid:

$$0 = cosx \cdot siny + cosy \cdot sinz + cosz \cdot sinx$$

lidinoid:

$$0 = 1 \cdot (sin2x \cdot sinz \cdot cosy + sin2y \cdot sinx \cdot cosz + sin2z \cdot siny \cdot cosx) - 1 \cdot (cos2x \cdot cos2y + cos2y \cdot cos2z + cos2z \cdot cos2x) + 0.3$$

schwarz p:

$$0 = cosx + cosy + cosz$$

split p:

$$0 = 1.1 \cdot (sin2x \cdot sinz \cdot cosy + sin2y \cdot sinx \cdot cosz + sin2z \cdot siny \cdot cosx) - 0.2 \cdot (cos2x \cdot cos2y + cos2y \cdot cos2z + cos2z \cdot cos2x) - 0.4 \cdot (cosx + cosy + cosz)$$

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